COHERENT BEAM-BEAM EFFECTS, THEORY & OBSERVATIONS

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Abstract

Current theoretical understanding of the coherent beam-beam effect as well as its experimental observations are discussed: conditions under which the coherent beam-beam modes may appear, possibility of their resonant interaction (coherent resonances), stability of beam-beam oscillations in the presence of external impedances. A special attention is given to the coherent beam-beam modes of finite length bunches: the synchro-betatron coupling is shown to provide reduction in the coherent tuneshift and - at the synchrotron tune values smaller than the beam-beam parameter - Landau damping by overlapping synchrotron satellites.

1 INTRODUCTION

Let us start with definition of the subject of the present report. By ***coherent*** effects we understand those arising from correlated in phase motion of particles (not necessarily with equal amplitudes as in a rigid-body motion).

There is a wider class of ***collective*** phenomena arising from mutual influence of two strong beams, e.g. the flip-flop effect, which are beyond the scope of this report. Here we just assume that a (quasi) equilibrium state does exist on a sufficiently long time scale.

The interest in the coherent beam-beam effect is twofold: it is useful in diagnostics of colliding beams but it is also a source of potential instability.

Though the coherent modes were routinely seen since the early days of e^+e^- colliders, there had been a long-standing issue of how the coherent tuneshifts are related to the beam-beam parameter until the work by K.Yokoya et al. [1]. In that paper an exhaustive answer was given on the basis of the Vlasov perturbation theory which was afterwards successfully used in the studies of coherent beam-beam resonances [2, 3], Landau damping by the beam-beam tunespread [4], diverse effects of the synchro-betatron coupling [5].

In the present report we give an overview of the Vlasov perturbation theory of coherent beam-beam effect, compare some of its results with numerical simulations and experimental observations.

2 COHERENT BEAM-BEAM MODES

Let us make a conventional choice of the generalized azimuth $\theta = s/R$ as the independent variable and describe particle motion with the help of action-angle variables

$$\underline{I} = \{I_x, I_y, I_s\}, \quad \psi_{x,y} = \psi_{x,y}^{(\text{original})} + \frac{\nu'_{x,y}}{\alpha_M R} z, \tag{1}$$

where the angle variables were renormalized to take into account chromaticity $\nu'_{x,y}$, α_M being the momentum compaction factor, R the average machine radius and z the longitudinal displacement w.r.t. the reference particle.

We choose the (quasi) equilibrium distribution function (see [6] for mathematical proof of existence) to be Gaussian:

$$F_0 = \frac{1}{(2\pi)^3 V} \exp\left(-\underline{\varepsilon}^{-1} \cdot \underline{I}\right) \tag{2}$$

$$\underline{\varepsilon} = \langle \underline{I} \rangle, \quad V = \varepsilon_x \varepsilon_y \varepsilon_s, \quad \underline{\varepsilon}^{-1} = (\varepsilon_x^{-1}, \varepsilon_y^{-1}, \varepsilon_s^{-1})$$

and study its small perturbations.

2.1 Vlasov perturbation theory

Evolution of the perturbation is governed by the Vlasov equation

$$\frac{\partial}{\partial\theta} F_1^{(k)} + \underline{\nu}^{(k)}(\underline{I}) \cdot \frac{\partial}{\partial \underline{\psi}} F_1^{(k)} = -F_0 \underline{\varepsilon}^{-1} \cdot \frac{\partial}{\partial \underline{\psi}} K_1^{(k)}(\underline{I}, \underline{\psi}; \theta) \tag{3}$$

where k = 1, 2 is the beam number, the r.h.s. describes the beam-wall and the beam-beam interaction; in the case of a finite bunch length the latter with the help of the ***synchro-beam transformation*** [7] can be presented in the form

$$K_1^{(k)}(\underline{I}, \underline{\psi}) = \sum_{IP} \frac{r_p N_{3-k}}{\gamma} \delta_p(\theta - \theta_{IP}) \int G^{(k)} F_1^{(3-k)}(\underline{I}', \underline{\psi}') d^3 I' d^3 \psi'.$$

By virtue of this transformation the interaction of a particle with the whole of the opposing bunch is lumped to the nominal interaction point, as a result the Green function explicitly depends on the momenta [5]:

$$G = -\ln\Big\{[x - x' + (\alpha + \frac{p_x + p'_x}{2})(z - z')]^2 + \\ + [y - y' + \frac{p_y + p'_y}{2}(z - z')]^2\Big\} \tag{4}$$

where α is half crossing angle. Coordinates in eq.(4) include the constant offset (if any) and the synchrotron part:

$$x = (-1)^{k-1} d_x / 2 + D_x \delta_p + \sqrt{2\beta_x I_x} \sin(\psi_x + \phi_x^{(k)}) \tag{5}$$

where $\phi^{(k)}_{x,y}$ is the betatron phase advance.

Eqs.(1,4,5) show how such factors as chromaticity, finite bunch length, crossing angle and dispersion enter the theory. Also, difference in intensity, bare lattice tunes and distribution in phase advances for the two beams can be taken into account.

It should be noted that by lumping the interaction in one point we exclude the possibility of the head particles in a finite-length bunch to talk to the tail particles via the opposing beam; thus we leave aside such important question as the beam-beam contribution to driving the head-tail instability.

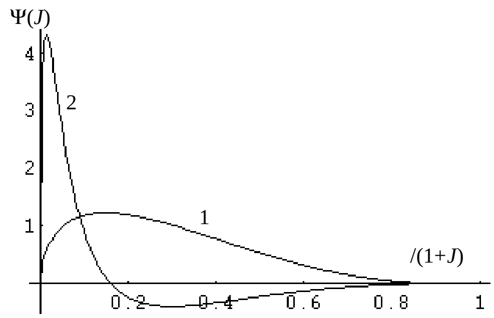

Figure 1. The first two discrete eigenmodes of horizontal oscillations in flat beams

2.2 Angle and radial modes

Performing Fourier expansion in the angle variables

$$\mathbf{f} = e^{\underline{\varepsilon}^{-1}\cdot\underline{I}/2}\begin{pmatrix}\sqrt{r_\xi}\,F_1^{(1)}\\ F_1^{(2)}\end{pmatrix} = \sum_{\underline{m}}\exp(i\underline{m}\cdot\underline{\psi})\,\mathbf{f}_{\underline{m}}(\underline{I},\theta) \qquad (6)$$

where $\underline{m} = \{m_x,\ m_y,\ m_s\}$ is the ***angle mode*** index, $r_\xi = N_1/N_2 \leq 1$, we cast the Vlasov equation into the form

$$i\frac{\partial}{\partial\theta}\mathbf{f}_{\underline{m}} = \sum_{\underline{m}'}\hat{A}_{\underline{m},\underline{m}'}\cdot\mathbf{f}_{\underline{m}'} \qquad (7)$$

with the matrix integral operators

$$\begin{aligned}\hat{A}_{\underline{m},\underline{m}'} &= \begin{pmatrix}\underline{m}\cdot\underline{\nu}^{(1)} & 0\\ 0 & \underline{m}\cdot\underline{\nu}^{(2)}\end{pmatrix}\delta_{\underline{m},\underline{m}'} + \\ &+ \underline{m}\cdot\underline{\varepsilon}^{-1}\sum_{IP}\frac{r_p}{\gamma}\sqrt{N_1N_2}\,\delta_{\mathrm{p}}(\theta-\theta_{IP})\begin{pmatrix}0 & \hat{G}^{(1)}_{\underline{m}\underline{m}'}\\ \hat{G}^{(2)}_{\underline{m}\underline{m}'} & 0\end{pmatrix}\end{aligned} \qquad (8)$$

For one-dimensional oscillations (e.g. horizontal for definiteness) $m_x = 1$ is usually referred to as the dipole mode, $m_x = 2$ as the quadrupole mode and so on. In fact each such mode presents a family of modes with different dependence on the action variables, called by B.Zotter the ***radial modes*** in contradistinction to the angle modes.

2.3 Discrete & continuous spectra

If the tunes are chosen so that for a given $\underline{m}$ the coherent resonance condition does not hold for any relatively low $\underline{m}'$,

$$\underline{m}\cdot\underline{\nu}^{(1)} + \underline{m}'\cdot\underline{\nu}^{(2)} \neq n\,, \qquad (9)$$

then we may consider the mode $\underline{m}$ uncoupled and formulate the eigenvalue problem for the corresponding family of radial modes:

$$\hat{A}_{\underline{m},\underline{m}}\cdot\mathbf{\Psi}_\lambda = \lambda\mathbf{\Psi}_\lambda \qquad (10)$$

For uncoupled modes the periodic δ-function in eq.(8) can be replaced with 1/2π.

Generally operator $\hat{A}_{\underline{m},\underline{m}}$ has mixed spectrum. Due to the first multiplicative part it necessarily has continuous spectrum with λ spanning the range of variation of the proper combinations of the incoherent tunes in both beams, $\underline{m}\cdot\underline{\nu}^{(1)} \cup \underline{m}\cdot\underline{\nu}^{(2)}$. The integral part of $\hat{A}_{\underline{m},\underline{m}}$ produces by itself purely discrete spectrum, however the total operator may or may not have discrete eigenvalues.

Eigenfunctions can be normalized so that

$$(\mathbf{\Psi}_\lambda, \mathbf{\Psi}_\mu) = \delta_{\lambda\mu} \qquad (11)$$

where the r.h.s. should be understood as the Kronecker symbol for λ belonging to the discrete spectrum, and as the Dirac δ-function if it belongs to continuum.

In the case of equal intensities and tunes the eigenmodes split into two classes: π-modes: $f^{(1)} = -f^{(2)} = f^{(-)}$, and Σ-modes: $f^{(1)} = f^{(2)} = f^{(+)}$.

The spectrum of dipole π-modes was found to be [1,3] (in units of the beam-beam parameter):

- round beams: $\lambda = 1.214$
- flat beams (horizontal): $\lambda = 1.330,\ 1.026,\ 1.002$
- flat beams (vertical): $\lambda = 1.239$

Fig.1 shows the first two radial modes of horizontal oscillations in flat beams as functions of $J = I_x/\varepsilon_x$.

For all geometries there is just one discrete Σ-mode with unshifted tune ($\lambda = 0$) corresponding to the rigid-body oscillations:

$$\Psi_0 = \sqrt{J_x}\,e^{-(J_x+J_y+J_s)/2} \qquad (12)$$

where $J_i = I_i/\varepsilon_i$.

In all these cases the spectra of both π- and Σ-modes include continuum (0, 1).

The ratio of the split in tunes of dipole π- and Σ-modes to the beam-beam parameter was dubbed the ***Yokoya factor***, *Y*.

There is a popular one-dimensional "slab" model in which the Yokoya factor was found to be as large as $Y = 1.5$ [6]. It should be stressed that there is a basic difference between this model and vertical oscillations in flat beams: in the latter case the problem is intrinsically two-dimensional [1], the vertical tune depends on the horizontal amplitude no matter how small the aspect ratio is. Higher dimensionality reduces coherence, hence smaller tuneshift than in the "slab" model.

Quadrupole π-mode also may have discrete eigenvalues [2,3], for horizontal oscillations in flat beams two such eigenvalues were found: $\lambda_1 = 2.044$, $\lambda_2 = 2.002$.

Important characteristics of the dipole eigenmodes are the ***spectral coefficients***

$$c_k(\lambda) = (\Psi_0, \Psi_\lambda^{(k)}), \quad k = 1,2 \qquad (13)$$

satisfying the relations

$$\int[c_1^2(\lambda) + c_2^2(\lambda)]d\lambda = 2, \quad \int c_1(\lambda)c_2(\lambda)d\lambda = 0 \qquad (14)$$

where the integral is understood in the Stieltjes sense: sum over the discrete eigenvalues and integral over the continuum.

Squares of coefficients (13) give the relative spectral weight of the mode in oscillations excited by a dipole kick at the corresponding beam [5].

For horizontal π-modes in flat beams $c_1 = -c_2 = 0.724$, 0.188, 0.064. Small values of the spectral coefficients of the second and third radial modes (and their proximity to the continuum boundary) explain why they were not seen in experiment.

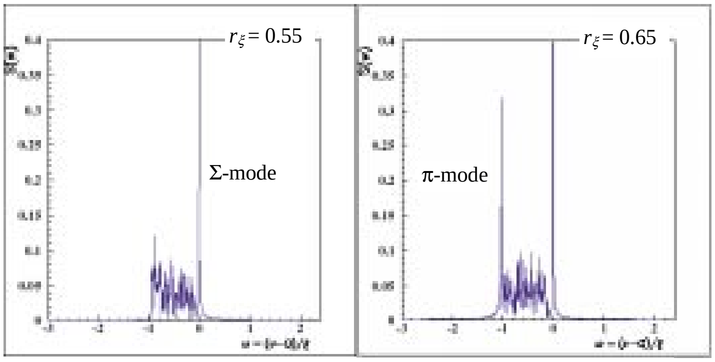

Figure 2. Spectra of oscillations in round *p-p* beams with the indicated values of intensity ratio.

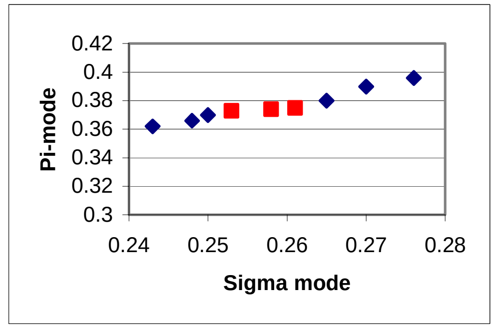

Figure 3. Spontaneous excitation of horizontal π-mode observed in LEP at 46GeV with beams colliding at 4 IPs (courtesy of K.Cornelis)

2.4 Transition from the weak-strong to the strong-strong regime

An important question is under what conditions the discrete eigenvalues may exist. There are a number of factors which affect coherence of oscillations, the basic one being the intensity ratio r_ξ.

It was shown that in the round beams the discrete mode emerges from the continuum at $r_\xi \approx 0.6$ [4]. Fig.2 presents results of simulations by the Hybrid Fast Multipole Method [8] which confirm this conclusion.

Another important factor is difference in tunes of the two beams [9]. It was found that the tunesplit $\geq Y\xi$ is necessary to damp both π and Σ discrete modes [3].

Discussion of these and other factors (and their possible interference) can be found in Ref.[5].

2.5 Experimental observations

Dedicated studies were performed at CESR for the vertical plane [10] and at Tristan accumulator ring for both transverse planes [1]. Measured values of the Yokoya factor coincide with theoretical values within a few percent.

The only observation of coherent beam-beam modes in hadron beams was made at RHIC [11]. There was also found a good agreement with theoretical predictions.

3 COHERENT RESONANCES

If the condition of a coherent resonance is met,

$$\underline{m}_1 \cdot \underline{\nu}^{(1)} + \underline{m}_2 \cdot \underline{\nu}^{(2)} = n \qquad (15)$$

coupling between the modes $\underline{m} = \underline{m}_1$ and $\underline{m}' = -\underline{m}_2$ in eq.(8) should be taken into account. If parity of m_{1x} and m_{2x} or of m_{1y} and m_{2y} is different, then respectively horizontal or vertical offset is needed for the beam-beam interaction to produce the coupling.

Analysis shows that this coupling may lead to instability only in the case

$$(\underline{m}_1 \cdot \underline{\varepsilon}^{-1})(\underline{m}_2 \cdot \underline{\varepsilon}^{-1}) > 0 \qquad (16)$$

Coherent beam-beam resonances were observed experimentally [12] and in simulation [13]. Fig.3 shows the measured dependence of the horizontal dipole π-mode tune on the Σ-mode tune in LEP at 46GeV. The red square data points mark the region of spontaneous excitation of the π-mode. This excitation was explained in [3] as a resonance of the dipole π-mode ($m_{1x} = 1$) and the quadrupole Σ-mode ($m_{2x} = 2$) in the presence of a moderate offset.

The possibility of such a resonance was confirmed in [13] by tracking with the use of the soft-Gaussian approximation for beam-beam kick. It was found however

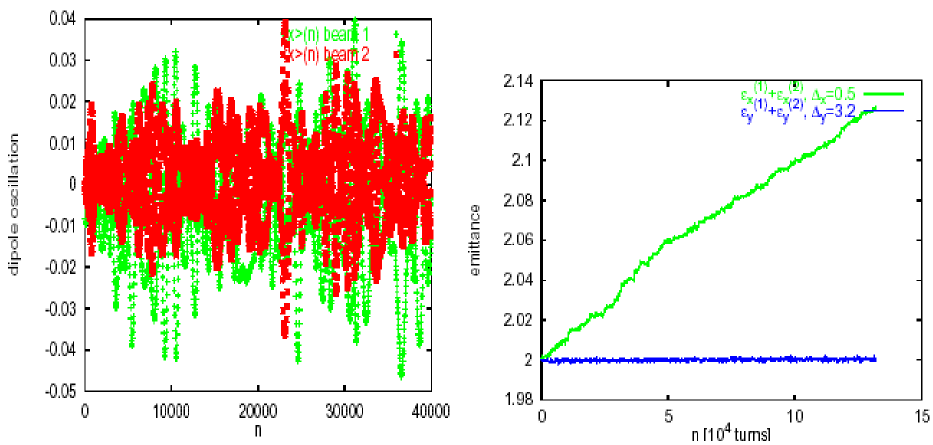

Figure 4. Tracking simulation of the dipole-quadrupole resonance at an offset of $0.3\sigma_x$ [13]. Left: center-of-mass oscillations in the two beams, right: combined horizontal emittance growth.

that the instability saturates at relatively small dipole amplitudes ($<0.04\sigma_x$) but at the expense of an unceasing emittance growth (Fig.4)

In absence of offsets higher order resonances (quadrupole-octupole in particular) can produce the emittance growth, but at a much lower rate [13].

4 FINITE BUNCH LENGTH EFFECTS

There are various sources of the Hamiltonian synchro-betatron coupling, we will consider here the betatron phase variation along the interaction region ("finite length effect") and chromaticity, the two in the case of short bunches combine in parameter (for horizontal oscillations)

$$\kappa = (\nu_x' / \alpha_M R - 1/\beta^*)\sigma_s \tag{17}$$

There are three distinct regimes depending on the ratio of the synchrotron tune to the beam-beam parameter [5]:

- Small beam-beam parameter (high ν_s).

The effect of coupling can be quantified by factor $\lambda_{||}$ (longitudinal eigenvalue) which can be extracted from the integral operator in the second term of operator (8); in the case $\kappa^2<<1$ it is [5]

$$\lambda_{||} = e^{-\kappa^2}\ \mathrm{I}_{m_s}(\kappa^2) \tag{18}$$

where $\mathrm{I}_m(x)$ is the modified Bessel function of order m.

Due to this factor the tunes of the π- and Σ-modes in finite-length bunches are shifted towards the center of the continuum, the Yokoya factor can be estimated as $Y \sim \lambda_{||} Y_0$ with Y_0 being the value for infinitely short bunches.

Eq.(18) holds for the synchrotron modes $m_s \neq 0$ as well and shows that the coherent contribution to their spectra is strongly suppressed, the tunes of both π and Σ synchrotron modes being determined by the average incoherent tunes (~ξ/2 for head-on collisions)

$$\nu_{m_s} \approx \overline{\nu_{\mathrm{inc}}} + m_s \nu_s \tag{19}$$

- Large beam-beam parameter (low ν_s).

In this limit $\lambda_{||} \approx 1$ so that the Yokoya factor is not affected, but the oscillations are not purely dipole, their phase varies along the bunch.

- Comparable values of the beam-beam parameter and the synchrotron tune.

The effect of synchro-betatron coupling is most dramatic in this case, the synchrotron sidebands of the continuum modes can overlap spectral lines of discrete π- and Σ-modes thus providing their Landau damping [5]. This prediction was confirmed by tracking in the soft-Gaussian approximation [14].

The described dependence of the coherent modes on the synchrotron tune can be compared with experimental results obtained at VEPP-2M [15]. Fig.5 shows the measured vertical tunes as functions of the beam-beam parameter at fixed value of the synchrotron tune $\nu_s = 0.007$ and $\sigma_s/\beta^* \approx 0.6$. At small values of the beam-beam parameter ($\xi \leq 0.005$/IP) the Yokoya factor appears as small as $Y \approx 0.65$. In the opposite limit ($\xi = 0.03$/IP) taking into account the dynamic focusing effect it can be estimated as $Y \approx 1.14$.

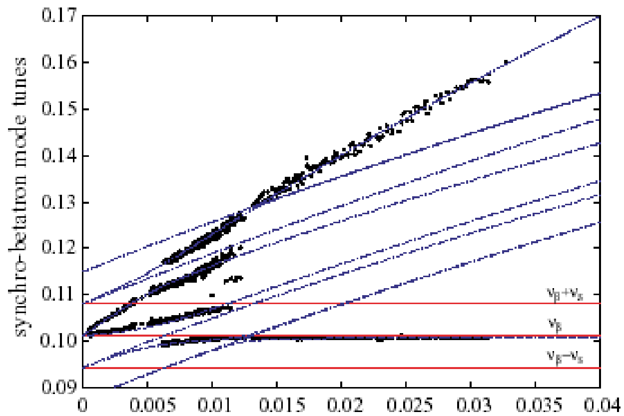

Figure 5. Measured tunes (dots) of vertical oscillations of one e^+ and one e^- bunches colliding at two IPs in VEPP-2M as functions of the beam-beam parameter/IP; $\nu_y = 3.101$, $\nu_s = 0.007$, $\sigma_s = 3.5$cm, $\beta^* = 6$cm.

Although numerically the Yokoya factor occurs smaller than expected in both limits, its qualitative behavior w.r.t. the ratio of the beam-beam parameter to the synchrotron tune is close to the prediction.

5 BEAM-BEAM EFFECT AND IMPEDANCE DRIVEN INSTABILITIES

Interplay of beam-beam and beam-wall interactions is the major reason for the continuing interest in coherent beam-beam effect.

5.1 *Landau damping of the beam-beam modes*

It was first suggested by J.Gareyte [16] that the large gaps between the coherent and incoherent tunes may switch off Landau damping in the strong-strong regime thus leaving the beams liable to instability.

As the further studies have shown, there is possibility to damp the discrete coherent modes by tunesplit and/or overlapping synchrotron sidebands.

The analytical theory of Landau damping by synchrotron sidebands was extended in [17] on the case of large bunch length, $\sigma_s \sim \beta^*$. Computed with its help (in the simplified case of flat beams at IP) beam-beam spectra in Tevatron at three values of chromaticity are presented in the upper row in Fig.6. When the chromaticity is close to the value $\nu_x' = 8$ which renders $\kappa = 0$ in eq.(17) the discrete modes are clearly seen, but are completely submerged into the continuum when the chromaticity is sufficiently far from this value.

The lower plot in Fig.6 demonstrates that in the case of unfavorable values of chromaticity Landau damping can be restored by splitting the bare lattice tunes in two beams by an amount $\geq \xi_x^{(\mathrm{pbar})}$ as discussed in Section 2.

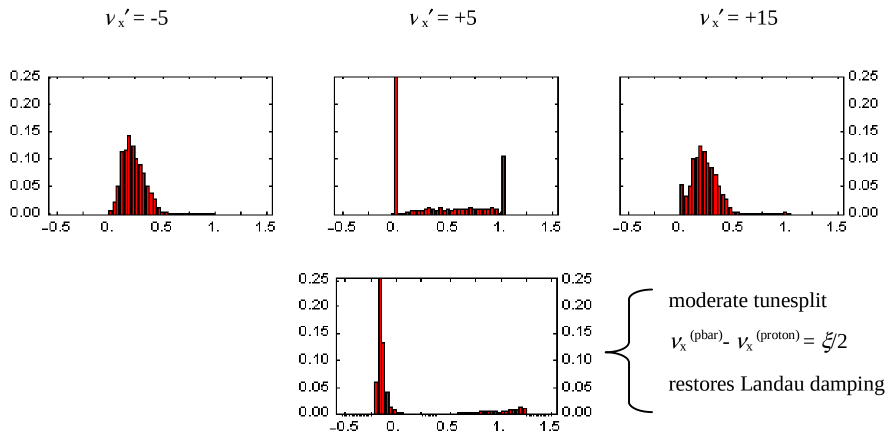

Figure 6. Effect of chromaticity and tunesplit on the beam-beam oscillations spectrum at the Tevatron upgrade parameters $\sigma_s/\beta^* = 50/35$, $\nu_s/\xi_x^{(pbar)} = 0.035$, $r_\xi = N_{pbar}/N_p = 1/2$, abscissa values in $\xi_x^{(pbar)} = 0.02$.

5.2 Aggravation of TMCI by LR interactions

It was observed at injection energy in LEP that the TMCI threshold in 8×8 operation was ~25% lower than in 1×1 case [18]. This reduction was caused mainly by the midarc long-range interactions where the beams were separated horizontally.

In the case of long-range interactions the coherent π-mode is shifted twice as much as the average incoherent tune and, according to eq.(19), the tunes of the synchrotron modes. In the result at some value of the beam current the tunes of the dipole and $m_S = -1$ π-modes in the plane of separation collide (Fig.7) creating potential for instability.

In the other plane (vertical in this case) the unstable situation is created by the $m_S = -1$ Σ-mode being shifted upwards to the dipole Σ-mode.

The instability itself was driven by the non-Hamiltonian coupling via the wake-fields, not by the beam-beam interaction.

However, there is still an open question whether the interaction of very long bunches can drive the head-tail instability. Some indication of such a possibility can be seen in the results of simulations with the ODYSSEUS code [19].

6 ACKNOWLEDGEMENTS

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